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INTENSIVE INJECTION OF FLUID INTO A HYPERSONIC STREAM FROM THE SURFACE OF A FINITE LENGTH PLATE

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Intensive fluid injection into a hypersonic stream from the surface of a finite length plate is considered. The injected fluid is assumed perfect and inviscid, and the flow in the injection region, separated from the main stream by a contact surface, is defined by approximate "thin layer" equations. A complete analytic solution of the problem is presented. It leads to the establishment of universal formulas in similarity variables that define the contact surface form and pressure distribution on the plate. These universal formulas are valid for any given pressure at the plate trailing edge.

Models of perfect incompressible liquid and perfect gas were used in [1 — 10] for solving the problem of flow in the region of injection of gases and combustible mixtures into supersonic streams. Transverse pressure gradients were neglected and equations of the thin layer were used. Theoretical and experimental investigations [4 — 7, 9 — 11] of flows with intensive injection from the surface of bodies of finite dimensions in a supersonic stream have shown that pressure at the trailing edge plays an important part in the formation of flow in the injection region. In [4, 5, 9] the problem of liquid and gas injection from the surface of a wedge into super- and hypersonic streams were solved by numerical integration of the thin layer equations. Integral equations were obtained for pressure at the plate surface in [6, 7, 10], when solving similar problems. The latter approach made it possible to obtain in [10] and in the present investigation complete analytic solutions of problems of super- and hypersonic flows over the layer of fluid injected from a plane surface. Analytic solutions were obtained for injection uniformly distributed over the length of the plate. Self-similar fluid injection conforming to power law was investigated in [3, 4, 8].

1. Statement of the problem. Let us consider the hypersonic flow of gas over a thin layer of liquid injected from the surface of a plane plate of finite length. The plate is parallel to the velocity vector of the oncoming stream and the liquid is injected uniformly over the whole length of the plate from the leading to the trailing edges in a direction normal to the plate. We use a Cartesian coordinate system with the x -coordinate on the plate surface and the origin at its leading edge. We assume the relative thickness of the injected liquid layer and the inclination of its streamlines to be small. We denote the order of that relative thickness by δ .

For the determination of pressure at the contact surface under hypersonic flow conditions we use the formula of tangent wedges

$$p = \frac{2}{\gamma+1}\rho_\infty U_\infty^2 \sin^2\alpha \tag{1.1}$$

or Newton's formula [12]

$$p = p^* \sin^2\alpha, \quad p^* = \frac{\rho_\infty U_\infty^2}{\gamma M_\infty^2}\left[\left(\frac{\gamma+1}{2}M_\infty^2\right)^{\gamma/(\gamma-1)} \times \left(\frac{\gamma+1}{2\gamma M_\infty^2-\gamma+1}\right)^{1/(\gamma-1)} - 1\right] \tag{1.2}$$

where ρ_∞, U_∞, M_∞, γ are, respectively, the density, velocity, Mach number, and adiabatic exponent of the oncoming stream, α is the angle between the tangent to the contact surface and the x-axis. The classic Newton's formula [12] is a particular case of formula (1.1) for $\gamma = 1$. Hence taking into account the smallness of inclination of the contact discontinuity surface, we can define the dimensionless pressure p' which is of order unity, by formulas

$$p = \frac{2}{\gamma+1}\rho_\infty U_\infty^2 \delta^2 p', \quad p = p^*\delta^2 p' \tag{1.3}$$

which correspond to formulas (1.1) and (1.2) for pressure.

We introduce in the injection region the dimensionless variables of order unity

$$x = lX, \quad y = \delta lY, \quad u = \frac{V_w}{\delta}u' \tag{1.4}$$
$$v = V_w v', \quad p = \rho_0\left(\frac{V_w}{\delta}\right)^2 p'$$

where l is the plate length, and ρ_0 and V_w are, respectively, the density and normal velocity of the injected fluid. The comparison of formulas (1.3) and (1.4), using expressions (1.1) and (1.2) for pressure at the contact surface yields for the relative thickness of the layer, respectively

$$\delta = \left[\frac{(\gamma+1)\rho_0 V_w^2}{2\rho_\infty U_\infty^2}\right]^{1/4}, \quad \delta = \left(\frac{\rho_0 V_w^2}{p^*}\right)^{1/4}$$

Note that the order of the relative thickness of the injected layer determined in [10] for moderate supersonic velocities of the oncoming stream differs from the obtained above for hypersonic velocities of flow; it is equal to the ratio of velocity heads to power $^1/_3$.

Using dimensionless variables (1.4) the following system of equations of thin layer in Mises variables:

$$\frac{\partial}{\partial X}\left(\frac{u'^2}{2}\right) + \frac{\partial p'}{\partial X} = 0, \quad \frac{\partial p'}{\partial \Psi} = 0, \quad \frac{\partial Y}{\partial \Psi} = \frac{1}{u'}$$
$$\frac{\partial \Psi}{\partial X} = -v', \quad \frac{\partial \Psi}{\partial Y} = u'$$

was obtained in [6] from Euler's equations in zero approximation with respect to δ^2.

Formulas (1.1) and (1.2) in zero approximation with respect to δ^2 yields

$$p' = (dY_s / dX)^2 \tag{1.5}$$

where Y_s is the dimensionless ordinate of the contact surface.

Integrating the system of equations of the thin layer we obtain for the contact surface ordinate the following expression [6]:

$$Y_s(X) = 2^{-1/2} \int_0^X [p'(\xi) - p'(X)]^{-1/2} d\xi$$

which together with (1.5) yields for the pressure on the plate the integral equation

$$\int_0^X \frac{d\xi}{\sqrt{p'(\xi) - p'(X)}} = \sqrt{2} \int_0^X \sqrt{p'(\xi)}\, d\xi, \quad 0 \leqslant X \leqslant 1 \tag{1.6}$$

To obtain a solution of Eq. (1.6) that would correspond to some pressure specified at the plate trailing edge it is necessary to add the boundary condition $p'(1) = p_0'$, where p_0' is the pressure at the trailing edge normalized in conformity with formulas (1.3). For function $X(p')$ we can obtain the following equation:

$$\int_{+\infty}^{p'} \frac{X'(q)\, dq}{\sqrt{q - p'}} = \sqrt{2} \int_{+\infty}^{p'} \sqrt{q} X'(q)\, dq \tag{1.7}$$

Integrals (1.7) have an infinite limit of integration, since at normal injection the pressure by virtue of (1.5) becomes infinite. Equation (1.7) is linear. Its solution $x_0(p')$ along the semiaxis $0 \leqslant p' < +\infty$ satisfies the boundary condition $x_0(0) = 1$ and is of a universal nature. Any solution of Eq. (1.7) that corresponds to some pressure p_0' is expressed in terms of that pressure by formula $X(p') = x_0(p') / x_0(p_0')$, $p_0' \leqslant p' < +\infty$.

2. Solution of the integral equation. Using the substitution for the variable $p' = e^{-t}$ with $-\infty < t < +\infty$ we transform Eq. (1.7) to

$$\int_{-\infty}^{t} \frac{e^{\tau/2} x_0'(e^{-\tau})\, d\tau}{(1 - e^{\tau - t})^{1/2}} = \sqrt{2} \int_{-\infty}^{t} e^{-\tau/2} x_0'(e^{-\tau})\, d\tau \tag{2.1}$$

$$x_0(e^{-t}) \to 1, \quad t \to +\infty$$

To make possible the application to Eq. (2.1) the two-sided Laplace transforms [13] we estimate the order of the unknown function decrease at $-\infty$. Let us assume that the inequality

$$|x_0'(e^{-t})| \leqslant \text{const}\ e^{(b+1/2)t}, \quad t < 0 \tag{2.2}$$

holds for some $b \geqslant 0$. From the left-hand side of Eq. (2.1) we, then, obtain by virtue of the convolution property that inequality (2.2) remains valid when $b+1$ is substituted for b and, consequently, is satisfied for any positive b. Assuming now that function $x_0'(e^{-t})$ increases exponentially at $+\infty$, we find that the operational correspondence

$$L[x_0'(e^{-t})] = \int_{-\infty}^{+\infty} x_0'(e^{-t}) e^{-zt}\, dt = h(z) \tag{2.3}$$

where L is the Laplace operator, has some convergence half-plane $\operatorname{Re} z > a$.

Applying the convolution theorem [13] to (2.1), by virtue of the operational correspondence [14]

$$L[(1-e^{-t})^{-1/2} H(t)] = \frac{\sqrt{\pi}\,\Gamma(z)}{\Gamma(z+1/2)}, \quad \operatorname{Re} z > 0$$

where $H(t)$ is the Heaviside unit function and $\Gamma(z)$ is a gamma function, for the determination of $h(z)$ according to Laplace we obtain the functional equation

$$h\left(z-\frac{1}{2}\right)\frac{\Gamma(z)\sqrt{\pi}}{\Gamma(z+1/2)} = \frac{\sqrt{2}}{z}\,h\left(z+\frac{1}{2}\right), \quad \operatorname{Re} z > \max\left(0, a+\frac{1}{2}\right) \tag{2.4}$$

Note that the solution of Eq. (2.4) has been determined with an accuracy to some arbitrary periodic function $\omega(z)$ of period unity.

Let us find the particular solution $h_0(z)$ of the equation. Taking the logarithms of both sides of Eq. (2.4) and differentiating twice the result, we obtain

$$\frac{d^2}{dz^2}\ln h_0\left(z+\frac{1}{2}\right) - \frac{d^2}{dz^2}\ln h_0\left(z-\frac{1}{2}\right) = \psi'(z+1) - \psi'\left(z+\frac{1}{2}\right) \tag{2.5}$$

where ($\psi(z)$ is the logarithmic derivative of the gamma function.

We introduce function $\theta(t)$ in conformity with the operational correspondence

$$\frac{d^2}{dz^2}\ln h_0(z) = L[\theta(t)] \tag{2.6}$$

Applying to (2.5) the inverse Laplace transform and using the operational correspondence [14]

$$L[-te^{-t/2}(1+e^{-t/2})^{-1} H(t)] = \psi'(z+1) - \psi'(z+1/2)$$

we obtain for function $\theta(t)$ the expression

$$\theta(t) = \frac{1-e^{-t/2}}{(1-e^{-t})^2}\,te^{-t}H(t) \tag{2.7}$$

Applying now the Laplace transform to (2.7), we obtain

$$\frac{d^2}{dz^2}\ln h_0(z) = \psi\left(z+\frac{1}{2}\right) - \psi(z) + \left(z+\frac{1}{2}\right)\psi'\left(z+\frac{1}{2}\right) - z\psi'(z)$$

By integrating this expression twice we obtain

$$\ln h_0(z) = \int_z^{z+1/2} \zeta\psi(\zeta)\,d\zeta + kz + q = \left(z+\frac{1}{2}\right)\ln\Gamma\left(z+\frac{1}{2}\right) - z\ln\Gamma(z) - \int_z^{z+1/2}\ln\Gamma(\zeta)\,d\zeta + kz + q \tag{2.8}$$

where k and q are constants of integration. We determine $k = 1/2 \ln\sqrt{\pi/2} - 1/2$ by substituting expression (2.8) into the logarithm of Eq. (2.4) and using the equality [15]

$$\int_0^1 \ln\Gamma(\zeta)\,d\zeta = \ln\sqrt{2\pi}$$

Moreover, as shown below, condition $x_0\ (e^{-t}) \to 1$ is satisfied, as $t \to +\infty$, when by appropriate selection of constant q we obtain $h_0(0) = 1$. We then have

$$q = \frac{1}{2}\ln\frac{2}{\sqrt{\pi}} + \frac{1}{2} + G$$

$$G = \int_0^{1/2} \ln\Gamma(\zeta+1)\,d\zeta = -0.0428536$$

where the definition of the constant G conforms to [16].

The asymptotics of function $h_0(z)$ at infinity can be obtained from (2.8) using the asymptotics of the gamma function [17]

$$\ln h_0(z) \sim \frac{z}{2}\ln z + kz + \frac{1}{8}\ln z + q - \frac{1}{8} - \frac{1}{48z} + \frac{1}{128z^2} + \ldots, \quad |z| \to \infty, \quad |\arg z| < \pi - \varepsilon, \quad \varepsilon > 0 \tag{2.9}$$

The general solution of Eq. (2.4) is of the form $h(z) = \omega(z)\, h_0(z)$. As a Laplace image, function $h(z)$ must be holomorphic in the half-plane $\operatorname{Re} z > a$ and approach zero at the ends of any straight line $\operatorname{Re} z = c,\ c > a$. Formula (2.8) makes it possible to conclude that only points $z = -m,\ -{}^1/_2 - m$ can be singular points of function $h_0(z)$. Hence function $\omega(z) = h(z) / h_0(z)$ is holomorphic for $\operatorname{Re} z > \max(a, -1)$. From this, owing to the periodicity of function $\omega(z)$ of period unity, follows its holomorphy throughout the complex plane. From (2.9) we find that on any straight line parallel to the imaginary axis and lying in the right-hand half-plane (m is integer)

$$|h_0(z)| > \text{const} \exp\left(-\frac{\pi}{4}|\operatorname{Im} z|\right)$$

The condition for $h(z)$ approaching zero implies that

$$|\omega(z)| \leqslant \text{const} \exp\left(\frac{\pi}{4}|\operatorname{Im} z|\right) \tag{2.10}$$

The entire periodic function of period unity which in the period band satisfies inequality (2.10) is constant. This statement can be simply proved by the method described in [18]. Thus we can set in what follows function $\omega(z)$ equal unity.

We introduce the universal function $y_0(p')$ which defined the contact surface ordinate

$$y_0(p') = \int_{+\infty}^{p'} x_0'(q)\sqrt{q}\,dq$$

Using Mellin's transform we obtain

$$x_0(e^{-t}) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} h_0(z)\, e^{zt}\frac{dz}{z} \tag{2.11}$$

$$y_0(e^{-t}) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} h_0\left(z+\frac{1}{2}\right) e^{zt}\frac{dz}{z}$$

$$c > 0, \quad -\infty < t < +\infty$$

Let us investigate the behavior of function $h_0(z)$ at points $z = -m$, $m = 1, 2, \ldots$. We have the following expansions in power series [15]:

$$\left(z+\frac{1}{2}\right)\psi\left(z+\frac{1}{2}\right) = \left(\frac{1}{2}-m\right)\psi\left(\frac{1}{2}-m\right) + \tag{2.12}$$
$$\left[\left(\frac{1}{2}-m\right)\psi'\left(\frac{1}{2}-m\right)+\psi\left(\frac{1}{2}-m\right)\right](z+m) +$$
$$\sum_{n=2}^{\infty}\left[(-1)^n(2^n-1)\zeta(n)+\left(m-\frac{1}{2}\right)(-1)^n(2^{n+1}-1)\zeta(n+1)+\right.$$
$$\left.S_2(n,m-1)-\left(m-\frac{1}{2}\right)S_2(n+1,m-1)\right](z+m)^n,$$
$$|z+m|<{}^1/_2$$

$$-z\psi(z) = -\frac{m}{z+m}+m\psi(m+1)+1+$$
$$[m\zeta(2)-\psi(m+1)+mS_1(2,m)](z+m)+$$
$$\sum_{n=2}^{\infty}[(-1)^{n+1}m\zeta(n+1)+(-1)^{n+1}\zeta(n)+$$
$$mS_1(n+1,m-1)-S_1(n,m-1)](z+m)^n. \quad |z+m|<1$$

$$\left(S_1(n,m)=\sum_{k=1}^{m}k^{-n}, \quad S_2(n,m)=\sum_{k=1}^{m}\left(k-\frac{1}{2}\right)^{-n}\right)$$
$$m = 1,2,\ldots$$

where $\zeta(n)$ is the Riemann zeta function.

The equality

$$\frac{d}{dz}\ln h_0(z) = \left(z+\frac{1}{2}\right)\psi\left(z+\frac{1}{2}\right) - z\psi(z) + k$$

after integration and involution yields

$$h_0(z) = p_m(z+m)^{-m}\exp\left[\sum_{n=1}^{\infty}a_n(z+m)^n\right], \quad |z+m|<1 \tag{2.13}$$

$$a_1 = 1-\frac{c}{2}+(2m-1)\ln 2+mS_1(1,m-1)-$$
$$\left(m-\frac{1}{2}\right)S_2(1,m-1)+k$$

$$a_2 = ({}^3/_4-m)\,\zeta(2)-\ln 2+(m/2)\,S_1(2,\ m-1)-$$
$$(m/2-{}^1/_4)\ S_2(2,\ m-1)+{}^1/_2\,S_2(1,\ m-1)-$$
$${}^1/_2\,S_1(1,\ m-1)$$

$$a_n = \frac{1}{n}\Big[mS_1(n, m-1) - S_1(n-1, m-3) + S_2(n-1, m-1) -$$
$$\left(m - \frac{1}{2}\right) S_2(n, m-1) + (-1)^{n-1}\zeta(n-1)(2^{n-1} - 2) +$$
$$(-1)^{n-1}\zeta(n)\left(2^n m - 2m + \frac{1}{2} - 2^{n-1}\right)\Big], \quad n = 3, 4, \ldots$$

where $c = 0.577215$ is Euler's constant and the constant of integration p_m can be directly determined from (2.4). Using the recurrent formula (2.4) m times we obtain

$$h_0(z-m) = h_0(z)\prod_{k=0}^{m-1}\sqrt{\frac{2}{\pi}}\,\frac{\Gamma(z-k)}{\Gamma(z-k+{}^1/_2)}$$

$$p_m = \lim_{z\to 0} z^m h_0(z-m) = \prod_{k=0}^{m-1}\frac{\sqrt{2}}{\pi}\,\frac{(2k-1)!!}{k!\,2^k}$$

in which allowance is made for $h_0(0) = 1$. As implied by (2.13), $h_0(z)$ has poles of order m at points $z = -m$. It can also be shown that at points $z = -{}^1/_2 - m$ function $h_0(z)$ has m zeros.

The asymptotic expansion (2.9) shows that Jordan's lemma is applicable to integrals (2.11) and the latter can be represented in the form of series in residues of integrands. For instance, for the first integrand of (2.11) we have the equality

$$h_0(z)\exp(zt)\frac{1}{z} = \frac{-p_m}{m(z+m)^m}\exp(-mt)\exp\left[\left(a_1 + t + \frac{1}{m}\right)\times\right. \tag{2.14}$$
$$\left.(z+m) + \sum_{n=2}^{\infty}\left(a_n + \frac{1}{nm^n}\right)(z+m)^n\right]$$
$$|z+m| < 1, \quad m = 1, 2, \ldots$$

which makes it possible to determine the residues at each pole $z = -m$. For this it is necessary to calculate the coefficients $(z+m)^{m-1}$ of the exponent expansion in the right-hand side of (2.14). The calculation of coefficients of the expansion of the exponent in power series is conveniently carried out using the recurrent formula (see [19]).

Hence functions $x_0(e^{-t})$, $y_0(e^{-t})$ can be represented in the form of the following series that are convergent for all t:

$$x_0(e^{-t}) = 1 + \sum_{m=1}^{\infty} e^{-mt}P_{m-1}(t)$$

$$y_0(e^{-t}) = h_0\left(\frac{1}{2}\right) + \sum_{m=1}^{\infty} e^{-(m+{}^1/_2)t}Q_{m-1}(t)$$

where $P_m(t)$, $Q_m(t)$ are polynomials of power m. Returning to variable p' we obtain

$$x_0(p') = 1 - \frac{\sqrt{2}}{\pi}p' + \sum_{m=2}^{\infty} p'^m P_{m-1}(-\ln p') \tag{2.15}$$

$$y_0(p') = 2^{1/4}\pi^{-3/4}\exp\left(\frac{3}{4} + 2G\right) - \frac{2\sqrt{2}}{3\pi}p'^{3/2} + \sum_{m=2}^{\infty} p'^{m+1/2} Q_{m-1}(-\ln p')$$

The asymptotics of functions $x_0(p')$, $y_0(p')$ when $p' \to +\infty$ can be obtained by applying the saddle-point method to integrals (2.11) [20].

We present here only the calculation of asymptotics of function $x_0(e^{-t})$. Asymptotic representation of the second universal function can be similarly derived. We introduce the notation

$$\alpha(z) = \ln h_0(z) - \ln z + zt$$

The saddle point ξ lies on the abscissa and is the positive root of equation

$$\alpha'(z) = 0, \quad t < 0 \tag{2.16}$$

In the conformity with (2.9) we have the asymptotic expansions

$$\alpha'(z) = \frac{1}{2}\ln z + t + k + \frac{1}{2} - \frac{7}{8z} + \frac{1}{48z^2} + O\left(\frac{1}{z^3}\right) \tag{2.17}$$

$$\alpha''(z) = \frac{1}{2z} + \frac{7}{8z^2} + O\left(\frac{1}{z^3}\right)$$

To determine the asymptotic representation of the positive root of Eq. (2.16) as $t \to -\infty$ we use Newton's iteration method [20], taking $\xi_0 = 2\pi^{-1}\exp(-2t)$ as the zero approximation. The third iteration for the root of Eq. (2.16) yields the expression

$$\xi = \xi_0 + \frac{7}{4} - \frac{151}{96\xi_0} + \frac{15}{4\xi_0^2} + O\left(\frac{1}{\xi_0^3}\right) \tag{2.18}$$

After substitution of (2.18) into (2.9) and (2.17) we obtain

$$\alpha(\xi) = -\frac{\xi_0}{2} - \frac{7}{8}\ln\xi_0 - \frac{1}{8} + q - \frac{151}{192\xi_0} + \frac{15}{16\xi_0^2} + O\left(\frac{1}{\xi_0^3}\right)$$

$$\alpha''(\xi) = \frac{1}{2\xi_0} + O\left(\frac{1}{\xi_0^3}\right)$$

We integrate the integral in (2.11) along the straight line $\mathrm{Im}\, z = \xi$ whose direction at the saddle point coincides with the line of steepest descent. The expansion of $\alpha(z)$ in Taylor series in the neighborhood of point ξ and substitution of the variable of integration $z = \xi + i\alpha''(\xi)\eta$ transforms (2.11) to

$$x_0(e^{-t}) = \frac{\exp(\alpha(\xi))}{\pi\alpha''(\xi)} \int_0^{\infty} \exp\left[-\frac{1}{2\alpha''}\eta^2 - \frac{\alpha^{IV}}{12(\alpha'')^3}\eta^4 + \frac{\alpha^{VI}}{360(\alpha'')^5}\eta^6 + \dots\right] \cos\left[\frac{1}{\alpha''}\left(\frac{\alpha'''}{6(\alpha'')^2}\eta^3 - \frac{\alpha^{V}}{120(\alpha'')^5}\eta^5 + \dots\right)\right] d\eta \tag{2.19}$$

The following equalities are valid:

$$\frac{\alpha'''(\xi)}{[\alpha''(\xi)]^2} = -2 + O\left(\frac{1}{\xi_0^2}\right), \quad \frac{\alpha^{IV}(\xi)}{[\alpha''(\xi)]^2} = 8 + O\left(\frac{1}{\xi_0^2}\right)$$
$$\frac{\alpha^{V}(\xi)}{[\alpha''(\xi)]^4} = -48 + O\left(\frac{1}{\xi_0^2}\right), \quad \frac{\alpha^{VI}(\xi)}{[\alpha''(\xi)]^2} = 384 + O\left(\frac{1}{\xi_0^2}\right)$$

In all of the above formulas only terms that are necessary for obtaining asymptotics of function $x_0(e^{-t})$ with an accuracy to $O(\xi_0^{-3})$ are retained. The Laplace method can be applied to integral (2.19) [20].

After computations we finally obtain

$$x_0(e^{-t}) \sim \frac{\xi^{-3/8}}{\sqrt{\pi}} \exp\left(-\frac{\xi_0}{2} - \frac{1}{8} + q\right)\left(1 + \frac{137}{192\xi_0} + \frac{107759}{73728\xi_0^2} + \ldots\right)$$
$$y_0(e^{-t}) \sim \frac{\xi_0^{-3/8}}{\sqrt{\pi}} \exp\left(-\frac{\xi_0}{2} - \frac{t}{2} - \frac{1}{8} + q\right) \times$$
$$\left(1 + \frac{233}{192\xi_0} - \frac{182831}{73728\xi_0^2} + \ldots\right)$$

Returning to variable p' we have

$$x_0(p') \sim 2^{1/8}\left(\frac{e}{\pi}\right)^{3/8} p'^{-3/4} \exp\left(-\frac{p'^2}{\pi} + G\right) \times \tag{2.20}$$
$$\left(1 + \frac{137\pi}{384p'^2} + \frac{107759\pi^2}{294912p'^4} + \ldots\right)$$
$$y_0(p') \sim 2^{1/8}\left(\frac{e}{\pi}\right)^{3/8} p'^{-1/4} \exp\left(-\frac{p'^2}{\pi} + G\right) \times$$
$$\left(1 + \frac{233\pi}{384p'^2} - \frac{182831\pi^2}{294912p'^4} + \ldots\right), \quad p' \to +\infty$$

Formulas (2.15) and (2.20) for universal functions make it possible to determine pressure distribution on the plate and the contact surface form.

The universal curves which define the distribution of the pressure coefficient $c_p = 2\delta^2 p'$ on the plate and the contact surface form $y_0 = y_0(x_0)$ are

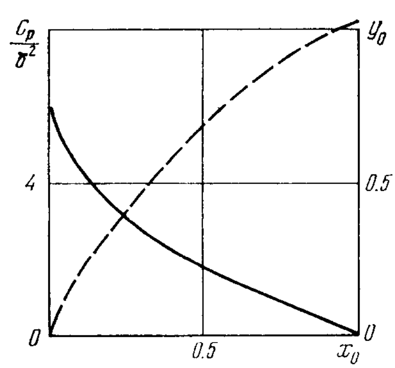

Fig. 1

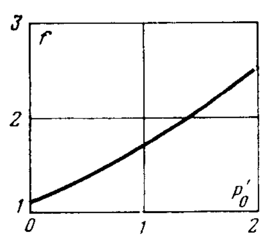

Fig. 2

shown in Fig. 1 in terms of dimensionless variables by solid and dash lines, respectively. For obtaining pressure distribution on the plate and the contact surface form that correspond to some pressure p_0' at the trailing edge the solid line in Fig. 1 is to be stretched along the x-coordinate, and the dash line also in the direction of the ordinate in proportion to $1 / x_0 (p_0')$, and take their parts contained in the interval $0 \leqslant X \leqslant 1$. The dependence of the resultant force F acting on the plate on pressure p_0' at the trailing edge is shown in Fig. 2. Since the considered here problem is plane, force F is calculated over a strip of the plate of unitary width. In Fig. 2 $f = \delta^2 F / (l \rho_0 V_w^2)$.

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